Cambright Solved Paper

Tags	2023 Additional Math CIE IGCSE May/June P2 V3
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Status	Done

1 **(a)** Solve the equation $\frac{|4x-5|}{7} = 1$. [2]

$|4x-5| = 7$

$(4x-5)^2 = 7^2$

$16x^2 - 40x + 25 = 49$

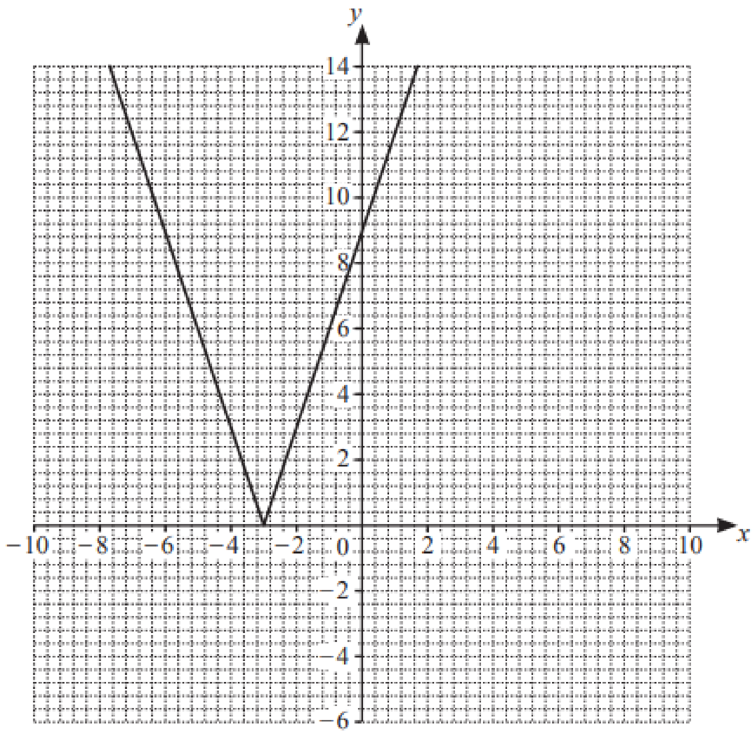
$16x^2 - 40x - 24 = 0$

$2x^2 - 5x - 3 = 0$

$(x-3)(2x+1) = 0$

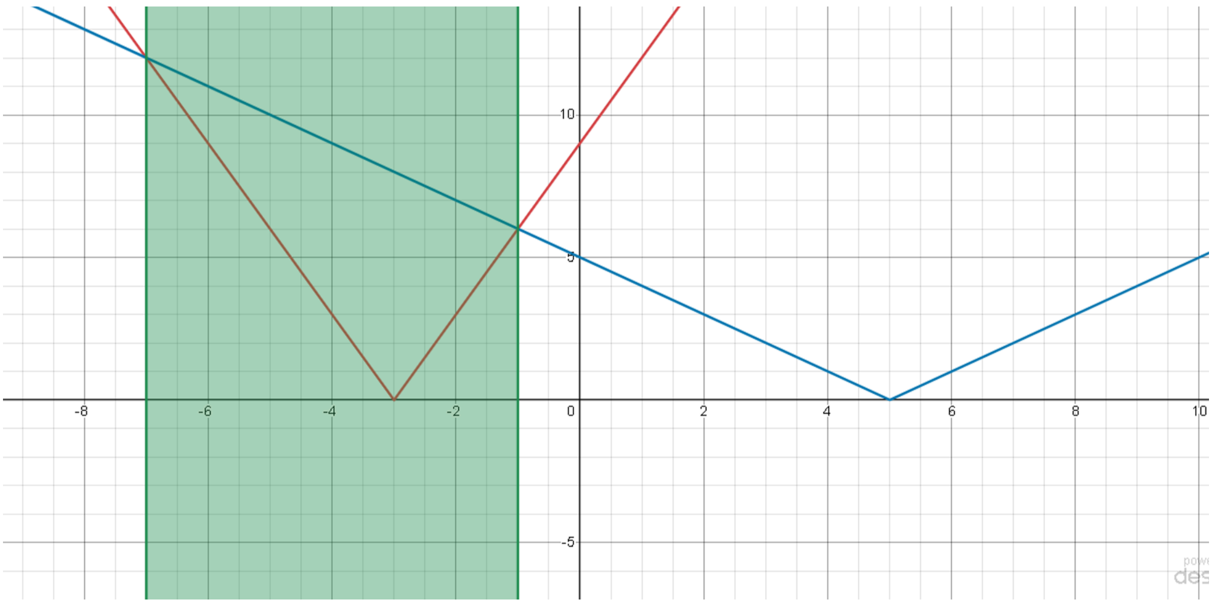
$x = 3 \; or \; x = -\frac{1}{2}$

(b)



The diagram shows the graph of $y = |3x+9|$.

By drawing a suitable graph on the same diagram, solve the inequality $|3x+9| \leqslant |x-5|$. [3]



https://www.desmos.com/calculator/ck5ftbfxpm

Here, we can see that $-7 \leq x \leq -1$ solves the inequality.

2 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Write the expression $\frac{\sqrt{98x^{12}}}{3+\sqrt{2}}$ in the form $(a\sqrt{b}+c)x^d$ where a, b, c and d are integers. [4]

$\sqrt{98} = 7\sqrt{2}$

$$\frac{7\sqrt{2}\,x^6}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}}$$
$$= \frac{21\sqrt{2}\,x^6-14x^6}{9-2}$$
$$= (\frac{21\sqrt{2}-14}{7})x^6$$
$$= (3\sqrt{2}-2)x^6$$

3 **(a)** Differentiate $\ln(x^3+3x^2)$ with respect to x, simplifying your answer. [2]

$$\frac{dy}{dx} = \frac{1}{x^3+3x^2} \times (3x^2+6x)$$
$$\frac{dy}{dx} = \frac{3x(x+2)}{x^2(x+3)}$$
$$\frac{dy}{dx} = \frac{3(x+2)}{x(x+3)}$$

(b) Hence find $\int \frac{x+2}{x(x+3)}\mathrm{d}x$. [2]

Since $\frac{dy}{dx} = \frac{3(x+2)}{x(x+3)}$, $\int \frac{3(x+2)}{x(x+3)} = \ln(x^3+3x^2)+c$

We want to remove the 3, so divide both sides by 3

$$\int \frac{(x+2)}{x(x+3)} = \frac{1}{3}\ln(x^3+3x^2)+c$$

4 The polynomial p is such that $p(x) = 2x^3 + 11x^2 + 22x + 40$.

(a) Show that $x = -4$ is a root of the equation $p(x) = 0$. [1]

$p(-4) = 2(-4)^3 + 11(-4)^2 + 22(-4) + 40$

$= -128 + 176 - 88 + 40$

$= 0$

(b) Factorise $p(x)$ and hence show that $p(x) = 0$ has no other real roots. [4]

$$
\begin{array}{r|l}
 & 2x^2 + 3x + 10 \\
\hline
x + 4 & 2x^3 + 11x^2 + 22x + 40 \\
 & 2x^3 + 8x^2 \\
\hline
 & 3x^2 + 22x \\
 & 3x^2 + 12x \\
\hline
 & 10x + 40 \\
 & 10x + 40 \\
\hline
 & 0
\end{array}
$$

$p(x) = (x + 4)(2x^2 + 3x + 10)$

If $2x^2 + 3x + 10$ has real roots, the discriminant will be greater than or equal to 0

$b^2 - 4ac = 3^2 - 4(2)(10)$

$9 - 80 = -71$

$x + 4$ is the only real root

5 (a) (i) A gardening group has 20 members. A committee of 6 members is to be selected. Anwar and Bo belong to the gardening group and at most one of them can be on the committee. How many different committees are possible? [2]

Case 1: Either Anwar or Bo is on the team, and the other is not. There are 18 people left to choose from and there are 5 spots left so ${}^{18}C_5 \times {}^{2}C_1 = 17136$

Case 2: Both Anwar and Bo are not on the team. There are 18 people left and 6 spots to choose,

so ${}^{18}C_6 = 18564$

Total = 35700

(ii) The gate for the garden has a lock with a 6-character passcode. The passcode is to be made from

Letters	G	A	R	D	E	N				
Numbers	0	1	2	3	4	5	6	7	8	9.

No character may be used more than once in any passcode.
Find the number of possible passcodes that have 4 letters followed by 2 numbers. [2]

${}^{6}P_4$ (6 letters, 4 letters to choose and arrange)

${}^{10}P_2$ (10 numbers, 2 to choose and arrange)

${}^{6}P_4 \times {}^{10}P_2 = 32400$

(b) (i) Given that $n \geqslant 4$, show that $(n-3) \times {}^{n}C_3 = 4 \times {}^{n}C_4$. [2]

$$\text{L.H.S} = (n-3) \times \frac{n!}{3!(n-3)!}$$

$$= \frac{n!}{3!(n-4)!}$$

Multiply by

$$\frac{4n!}{4(3!)(n-4)!} = \frac{4n!}{4!(n-4)!}$$

$$\text{R.H.S} = 4 \times \frac{n!}{4!(n-4)!}$$

$$= \frac{4n!}{4!(n-4)!}$$

So both sides are equal

(ii) Given that $^nC_3 = 5n$, where $n \geqslant 3$, show that n satisfies the equation $n^2 - 3n - 28 = 0$. Hence find the value of n. [4]

$$\frac{n!}{3!(n-3)!} = 5n$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{6\cancel{(n-3)!}} = 5n$$

$$n(n^2 - 3n + 2) = 30n$$

$$n^2 - 3n + 2 = 30$$

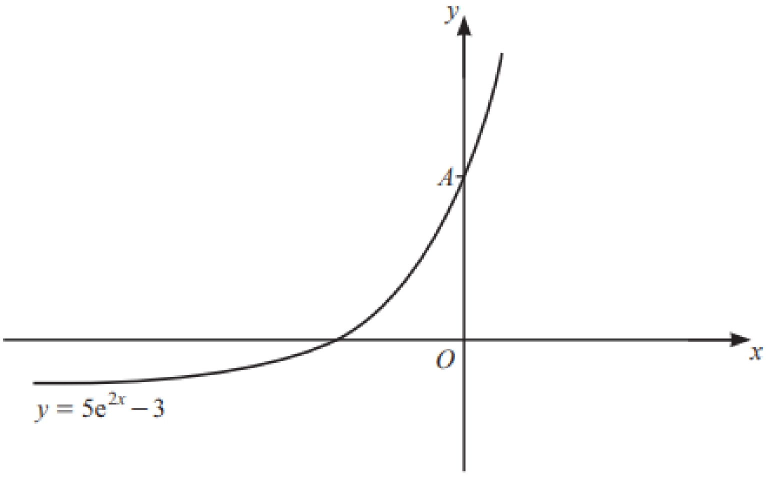
$$n^2 - 3n - 28 = 0$$

$$(n-7)(n+4) = 0$$

$$n = 7 \; or \; n = -4$$

n must be positive so $n = 7$

6



The diagram shows the curve $y = 5e^{2x} - 3$. The curve meets the y-axis at the point A. The tangent to the curve at A meets the x-axis at the point B. Find the length of AB. [6]

First find A, or the y-intercept when x = 0

$$x = 0 \rightarrow y = 5e^0 - 3$$

$$y = 5 - 3 = 2$$

Now, we know A (0, 2)

Now finding the gradient of the tangent

$$\frac{dy}{dx} = 10e^{2x}$$

$x = 0 \rightarrow \frac{dy}{dx} = 10e^0$

$\frac{dy}{dx} = 10$

Equation of tangent line,

$y = 10x + 2$ (Since the gradient is 10 and the y-intercept is 2)

B is the x-intercept because it is located on the x-axis.

$y = 0 \rightarrow 0 = 10x + 2$

$10x = -2$

$x = -\frac{1}{5}$

$\text{Length} = \sqrt{(-\frac{1}{5} - 0)^2 + (0 - 2)^2}$

$\text{Length} = 2.01$

7 Variables x and y are such that $y = \frac{4x^3 + 2\sin 8x}{1-x}$. Use differentiation to find the approximate change in y as x increases from 0.1 to $0.1 + h$, where h is small. [6]

$\frac{dy}{dx} = \frac{gf' - fg'}{g^2}$

$f = 4x^3 + 2\sin 8x,\ f' = 12x^2 + 16\cos 8x$

$g = 1 - x,\ g' = -1$

$\frac{dy}{dx} = \frac{(1-x)(12x^2 + 16\cos 8x) - (4x^3 + 2\sin 8x)(-1)}{(1-x)^2}$

$x = 0.1 \rightarrow \frac{dy}{dx} = \frac{(1-0.1)(12(0.1)^2 + 16\cos 8(0.1)) - (4(0.1)^3 + 2\sin 8(0.1))(-1)}{(1-0.1)^2}$

-

$\frac{dy}{dx} = 14.295$ (Make sure you use radians, since there is a trigonometric function)

$\frac{\delta y}{\delta x} \approx \frac{dy}{dx}$

$\delta y = 14.295 \times \delta x$

$\delta y = 14.295h$

8 **(a)** The functions f and g are defined by

$$\mathrm{f}(x) = \sec x \qquad \text{for } \frac{\pi}{2} < x < \frac{3\pi}{2}$$

$$\mathrm{g}(x) = 3(x^2 - 1) \qquad \text{for all real } x.$$

(i) Find the range of f. [1]

(ii) Solve the equation $\mathrm{f}^{-1}(x) = \frac{2\pi}{3}$. [3]

(i) Substitute the domain values in $f(x)$ and you will get a math error because the graph of cos is 0 at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, and the values between are negative.

$\therefore f \leq -1$

(ii) $f^{-1}(x) = \frac{2\pi}{3}$

$x = f(\frac{2\pi}{3})$

$x = \sec\frac{2\pi}{3}$

$x = \frac{1}{\cos\frac{2\pi}{3}}$

$x = -2$

(iii) Given that gf exists, state the domain of gf. [1]

(iv) Solve the equation $\mathrm{gf}(x) = 1$. [5]

(iii) Because f is part of the equation, simply use the domain of f

$$\frac{\pi}{2} < x < \frac{3\pi}{2}$$

(iv) $gf(x) = g(\sec x)$

$$= 3(\sec^2 x - 1)$$

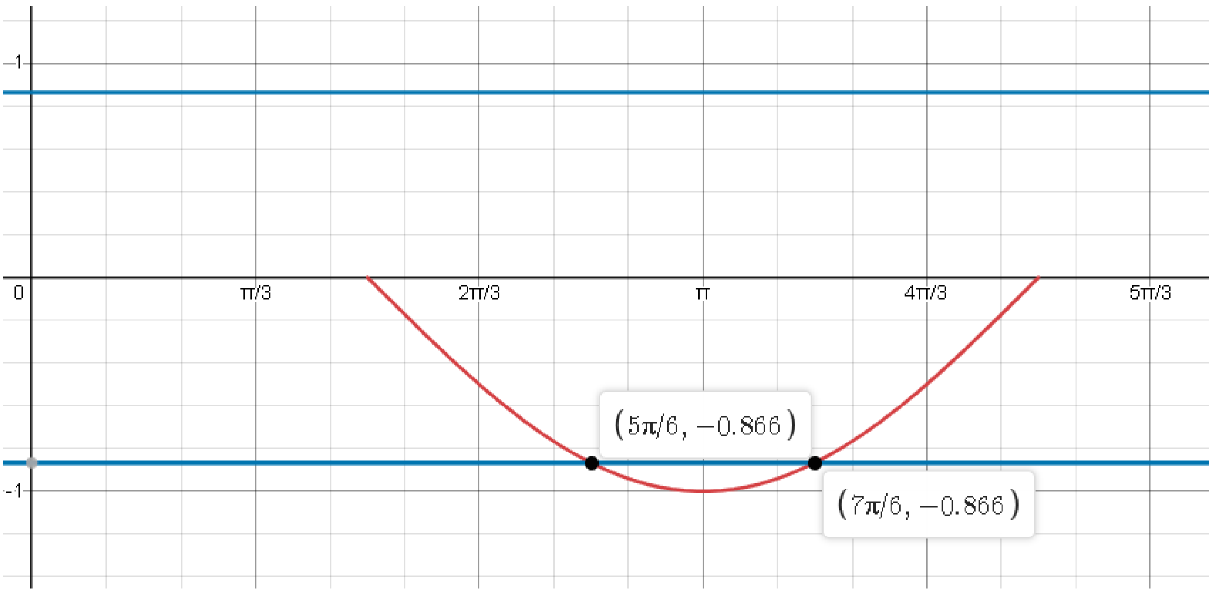
$$= 3(\frac{1}{\cos^2 x - 1})$$

$$3(\frac{1}{\cos^2 x} - 1) = 1$$

$$\frac{1}{\cos^2 x} - 1 = \frac{1}{3}$$

$$\cos^2 x = \frac{4}{3}$$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$



https://www.desmos.com/calculator/5wbpxf6ibo

Drawing a rough sketch + using our calculator, we find our 2 values

$x = \frac{5\pi}{6}$, $x = \frac{7\pi}{6}$

(b) The function h is defined by $h(x) = \ln(4 - x)$ for $x < 4$. Sketch the graph of $y = h(x)$ and hence sketch the graph of $y = h^{-1}(x)$. Show the position of any asymptotes and any points of intersection with the coordinate axes. [4]

x-intercept : $y = 0 \rightarrow 0 = \ln(4 - x)$

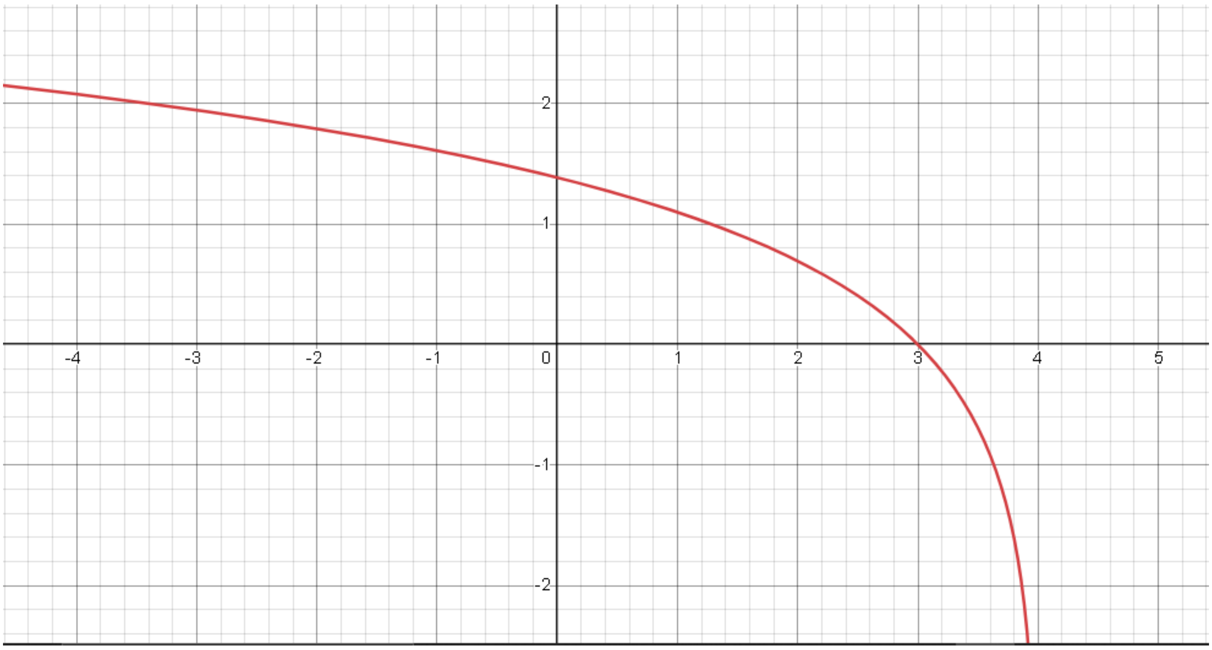
$e^0 = 4 - x$

$x = 3$

y-intercept: $x = 0 \rightarrow y = \ln 4$

$y = 1.386$

Asymptote: $x = 4$



https://www.desmos.com/calculator/fammzrpdn3

Label the interception points and asymptote too!

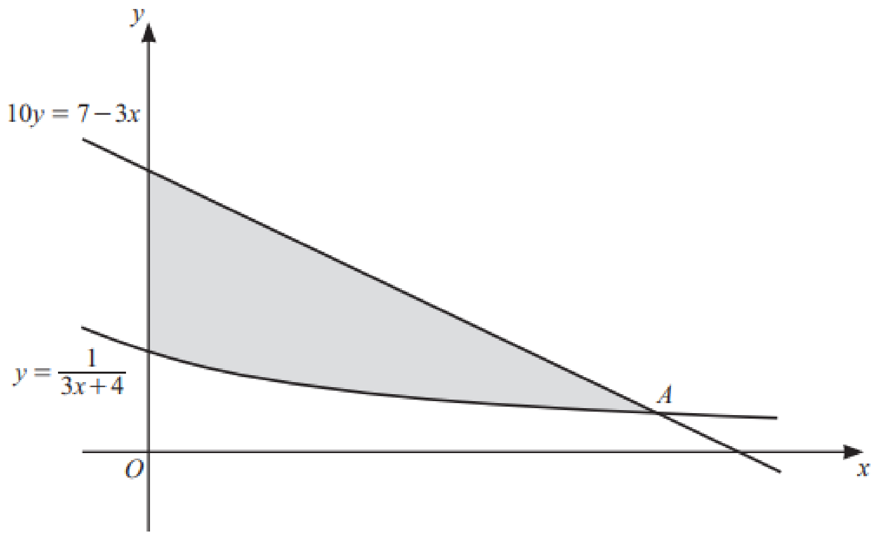
9 **(a)** Show that $\int_1^8 \frac{x+4}{\sqrt[3]{x}}\mathrm{d}x = 36.6$. [3]

$$\frac{x+4}{\sqrt[3]{x}} = \frac{x}{x^{\frac{1}{3}}} + 4x^{-\frac{1}{3}}$$
$$= x^{\frac{2}{3}} + 4x^{-\frac{1}{3}}$$

$$\int x^{\frac{2}{3}} + 4x^{-\frac{1}{3}}\ dx = \frac{3}{5}x^{\frac{5}{3}} + \frac{3}{2} \times 4x^{\frac{2}{3}}$$
$$= \frac{3}{5}x^{\frac{5}{3}} + 6x^{\frac{2}{3}}$$

$$[\frac{3}{5}x^{\frac{5}{3}} + 6x^{\frac{2}{3}}]_1^8 = [\frac{3}{5}8^{\frac{5}{3}} + 6 \times 8^{\frac{2}{3}}] - [\frac{3}{5}1^{\frac{5}{3}} + 6 \times 1^{\frac{2}{3}}]$$
$$= 43.2 - 6.6$$
$$= 36.6$$

(b)



The diagram shows part of the line $10y = 7 - 3x$ and part of the curve $y = \frac{1}{3x+4}$.
The line and curve intersect at the point A. Verify that the y-coordinate of A is 0.1 and calculate the area of the shaded region. [8]

To verify that y-coordinate of A is 0.1,

$$y = 0.1 \rightarrow \frac{1}{3x+4} = 0.1 \rightarrow \frac{10}{3x-4} = 1$$

$$3x + 4 = 10$$

$$3x = 6$$

$$x = 2$$

$$y = 0.1 \rightarrow 10(0.1) = 7 - 3x$$

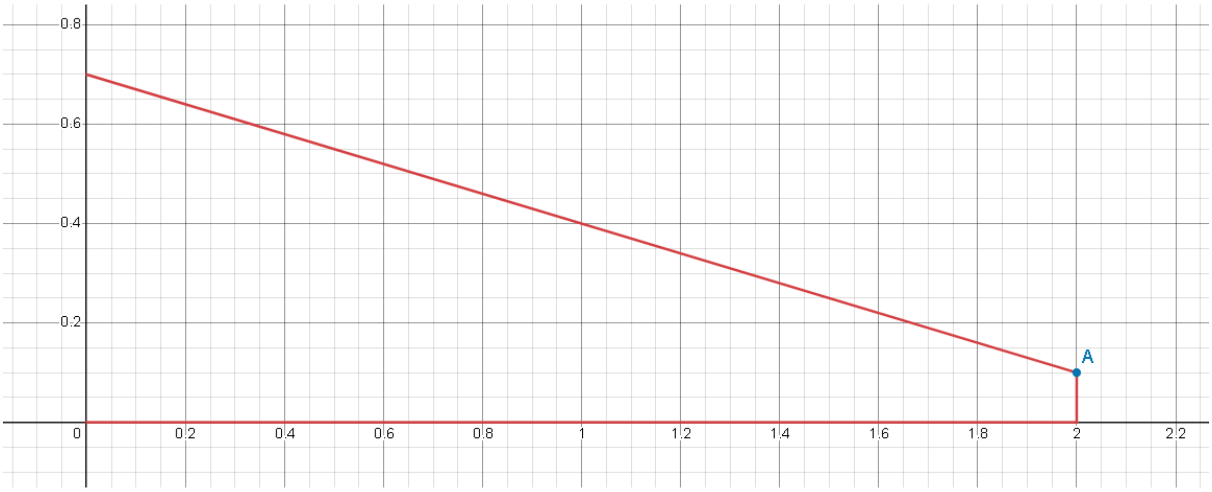
$$1 = 7 - 3x$$

$$-6 = -3x$$

$$x = 2$$

So, A does have a y-coordinate of 0.1!

Area of shaded region is the line's area minus the curve's area



https://www.desmos.com/calculator/vqi84m7j2v

We can see that if we draw a line from A to the x-axis, we get a trapezium

$$\text{Line's Area} = \frac{a+b}{2} \times b$$

We can take a = 0.1 and b as the y-intercept, which would be 0.7

$$= \frac{0.1+0.7}{2} \times 2$$

$$= 0.8$$

$$\text{Curve's Area} = \int_0^2 \frac{1}{3x+4}\,dx$$

$$let\ u = 3x + 4,\ \frac{du}{dx} = 3 \rightarrow dx = \frac{du}{3}$$

$$\text{Curve's Area} = \int_0^2 \frac{1}{u}\,\frac{du}{3}$$

$$= [\frac{1}{3}\ln u]_0^2$$

$$= [\frac{1}{3}\ln 3x + 4]_0^2$$

$$= [\frac{1}{3}\ln 3(2) + 4] - [\frac{1}{3}\ln 3(0) + 4]$$

$$= 0.305$$

$$\text{Shaded Area} = 0.8 - 0.305 = 0.495\ unit^2$$

10 An arithmetic progression, A, has first term a and common difference d.
The 2nd, 14th and 17th terms of A form the first three terms of a convergent geometric progression, G, with common ratio r.

(a) **(i)** Given that $d \neq 0$, find two expressions for r in terms of a and d and hence show that $a = -17d$. [6]

The given 3 terms of A can be written as:

$a + d,\ a + 13d,\ a + 16d$

Since these 3 terms are also the first 3 terms of G,

$\frac{a+13d}{a+d}\ and\ \frac{a+16d}{a+13d} = r$

$\therefore \frac{a+13d}{a+d} = \frac{a+16d}{a+13d}$

$a^2 + 16ad + ad + 16d^2 = a^2 + 13ad + 13ad + 169d^2$

$-9ad = 153d^2$

$-a = 17d$

$a = -17d$

If we put $a = -17d$ in $-9ad = 153d^2$,

$-9(-17d)d = 153d^2$

$153d^2 = 153d^2$

Therefore, $a = -17d$ is correct

(ii) Find the value of r. [2]

Using part i

$\frac{a+13d}{a+d}$, $a=-17d$

$= \frac{-17d+13d}{-17d+d}$

$= \frac{-4d}{-16d}$

$= \frac{1}{4}$

$\therefore r = \frac{1}{4}$

(b) The first term of the geometric progression, G, is q and the sum to infinity is $\frac{256}{3}$.

Find the sum of the first 20 terms of the **arithmetic** progression, A. [7]

The first term, which is $a+d$, or $-16d$, is q

$S_\infty = \frac{q}{1-r}$

$\frac{256}{3} = \frac{q}{1-0.25}$

$\frac{q}{0.75} = \frac{256}{3}$

$q = 64$

$\therefore -16d = q$

$d = \frac{64}{-16} \rightarrow d = -4$

$S_{20} = \frac{20}{2}\{2a + (20-1)d\}$

Sub $a = -17d$ (68) and $d = -4$

$S_{20} = 10\{2(68) + (19)(-4)\}$

$S_{20} = 10\{136 - 76\}$

$S_{20} = 10(60)$

$S_{20} = 600$

Additional notes

Websites and resources used:

- Desmos graphing calculator

If you find any errors or mistakes within this paper, please contact us and we will fix them as soon as possible.